Chapter 8
Geomechanical Analyses

Understanding rock mass response to tunnel and shaft construction is necessary to assess opening stability and opening support requirements. Several approaches of varying complexity have been developed to help the designer understand rock mass response. The methods cannot consider all aspects of rock behavior, but are useful in quantifying rock response and providing guidance in support design.

8-1. General Concepts

a. Stress/strain relationships.

(1) *Elastic parameters.*

(a) Elasticity is the simplest and most frequently applied theory relating stress and strain in a material. An elastic material is one in which all strain is instantaneously and totally recoverable on the removal of the stress. The theory of elasticity idealizes a material as a linear elastic, isotropic, homogeneous material.

(b) The stress/strain relationship for rock can sometimes be idealized in terms of a linear elastic isotropic material. In three dimensions, for an isotropic homogeneous elastic material subject to a normal stress σ_x in the x direction, the strains in the x, y, and z directions are:

$$\varepsilon_x = \sigma_x/E \qquad \varepsilon_y = \varepsilon_z = -\,v\,.\,\sigma_x/E$$

where

ε_x = applied stress in x-direction

v = Poisson's Ratio

E = modulus of elasticity

Since the principle of superposition applies, the stress/strain relationships in three dimensions are:

$$\varepsilon_x = (\,\sigma_x - v\,(\sigma_y + \sigma_z))/E$$

$$\varepsilon_y = (\,\sigma_y - v\,(\sigma_z + \sigma_x))/E$$

$$\varepsilon_z = (\,\sigma_y - v\,(\sigma_y + \sigma_x))/E$$

(c) For a competent rock that is not linear elastic, the stress/strain relationship can be generalized in the form of a curve with an increasing slope at low stress levels (related to closing of microcracks), an approximately linear zone of maximum slope over its midportion, and a curve of decreasing slope at stress levels approaching failure. In order to apply elastic theory to such rocks, it is necessary to define an approximate modulus of elasticity. The different methods available for defining this modulus of elasticity are as follows:

- Tangent modulus (E_T) to a particular point on the curve, i.e., at a stress level that is some fixed percentage (usually 50 percent) of the maximum strength.

- Average slope of the more-or-less straight line portion of the stress/strain curve.

- Secant modulus (E_s) usually from zero to some fixed percentage of maximum strength.

(d) Since the value of Poisson's Ratio is greatly affected by nonlinearities in the axial and lateral stress-strain curves at low stress levels, ASTM suggests that the Poisson's Ratio is calculated from the equation:

$$v = slope\ of\ axial\ curve/slope\ of\ lateral\ curve$$

(e) For most rocks, Poisson's Ratio lies between 0.15 and 0.30. Generally, unless other information is available, Poisson's Ratio can be assumed as 0.25. The modulus of elasticity varies over a wide range. For crude estimating purposes, the modulus of elasticity is about 350 times the uniaxial compressive strength of a rock (Judd and Huber 1961).

(f) Establishing values for elastic parameters that apply in the field takes judgment and should be made on a case-by-case basis. For a strong but highly jointed rock mass, a reduction in the value of E from the laboratory values of an order of magnitude may be in order. On the other hand, when testing very weak rocks (uniaxial compressive strength less than 3.5 MPa (500 psi)), sample disturbance caused by the removal of the rock sample from the ground may introduce defects that result in reduced values for the laboratory-determined modulus. For critical projects it is advisable to use field tests to determine the in situ deformability of rock.

(2) *Nonelastic parameters.* Many rocks can be characterized as elastic without materially compromising the

analysis of their performance. Where the stresses are sufficiently large that a failure zone develops around the tunnel, elastoplastic analyses are available for analyzing the stresses and strains. However, for some rocks such as potash, halite, and shales, time-dependent or creep movements may be significant and must be taken into account when predicting performance. Chabannes (1982) has established the time-dependent closure based on a steady-state creep law. Lo and Yuen (1981) have used rheological models to develop a design methodology for liner design that has been applied to shales. Time-dependent relationships are difficult to characterize because of the difficulty selecting rock strength parameters that accurately model the rock mass.

(3) *Rock strength.* Rock material is generally strong in compression where shear failure can occur and weak in tension. Failure can take the form of fracture, in which the material disintegrates at a certain stress, or deformation beyond some specific strain level. Rocks exhibit a brittle-type behavior when unconfined, but become more plastic as the level of confinement increases. Conditions in the field are primarily compressive and vary from unconfined near the tunnel walls to confined some distance from the tunnel. The strength of a rock is affected not only by factors that relate to its physical and chemical composition such as its mineralogy, porosity, cementation, degree of alteration or weathering, and water content, but also by the method of testing, including such factors as sample size, geometry, test procedure, and loading rate.

(4) *Uniaxial compressive strength.*

(a) The uniaxial or unconfined compressive strength is the geotechnical parameter most often quoted to characterize the mechanical behavior of rock. It can be misleading since field performance often depends on more than just the strength of an intact sample, and this value is subject to a number of test-related factors that can significantly affect its value. These factors include specimen size and shape, moisture content, and other factors. Uniaxial compressive strength usually should not be considered a failure criterion but rather an index that gives guidance on strength characteristics. It is most useful as a means for comparing rocks and classifying their likely behavior.

(b) The compressive strength of a rock material is size dependent, with strength increasing as specimen size decreases. It is useful to adjust the compressive strength values to take into account the size effect. An approximate relationship between uniaxial compressive strength and specimen diameter that allows comparison between samples is as follows:

$$\sigma_c = \sigma_{c50}\ (50/d)^{0.18}$$

where

σ_{c50} = compressive strength for a 50-mm- (2-in.-) diam sample

d = sample diameter (Hoek and Brown 1980)

(c) The compressive strength of a rock material often decreases when the rock is immersed in water. The reduced stresses may be due to dissolution of the cementation binding the rock matrix or to the development of water pressures in the interconnected pore space.

(5) *Tensile strength.* For underground stability, the tensile strength is not as significant a parameter as the compressive strength for rocks. Generally, tensile rock strength is low enough that when rock is in tension, it splits and the tensile stresses are relieved. As a rule of thumb, the tensile strength of rock material is often taken as one-tenth to one-twelfth of the uniaxial compressive strength of the intact rock. In jointed rocks, the jointing may very well eliminate the tensile strength of the rock mass, in which case the in situ rock should be considered as having zero tensile strength. Values of tensile strength and other geotechnical parameters of some intact rocks are given in Table 8-1.

(6) *Mohr-Coulomb failure criterion.*

(a) The Mohr-Coulomb failure criterion is most often applied to rock in the triaxial stress state. This criterion is based on (1) rock failure occurring once the shear stress on any plane reaches the shear strength of the material, (2) the shear strength along any plane being a function of the normal stress G_n on that plane, and (3) the shear strength being independent of the intermediate principal stress. The general form of the normal stress versus shear stress plot is shown in Figure 8-1. As an approximation over limited ranges of normal stress, the shear stress is defined as a linear relationship of the normal stress as follows:

$$\tau = c + \sigma_n \times \tan \phi$$

where

τ = shear strength

σ_n = applied normal stress

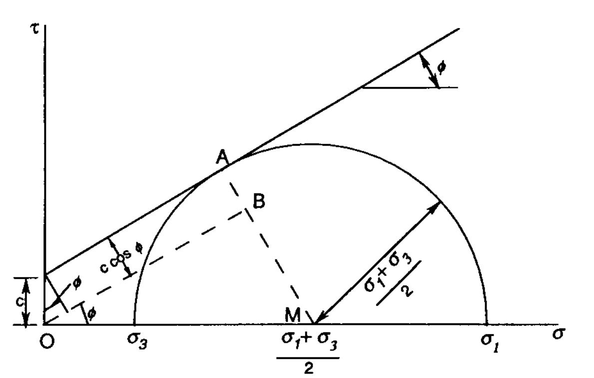
Table 8-1
Geotechnical Parameters of Some Intact Rocks (after Lama and Vutukuri 1978)

Rock Type	Location	Density Mg/m^3	Young's Modulus, GPa	Uniaxial Compressive Strength, MPa	Tensile Strength MPa
Amphibolite	California	2.94	92.4	278	22.8
Andesite	Nevada	2.37	37.0	103	7.2
Basalt	Michigan	2.70	41.0	120	14.6
Basalt	Colorado	2.62	32.4	58	3.2
Basalt	Nevada	2.83	33.9	148	18.1
Conglomerate	Utah	2.54	14.1	88	3.0
Diabase	New York	2.94	95.8	321	55.1
Diorite	Arizona	2.71	46.9	119	8.2
Dolomite	Illinois	2.58	51.0	90	3.0
Gabbro	New York	3.03	55.3	186	13.8
Gneiss	Idaho	2.79	53.6	162	6.9
Gneiss	New Jersey	2.71	55.2	223	15.5
Granite	Georgia	2.64	39.0	193	2.8
Granite	Maryland	2.65	25.4	251	20.7
Granite	Colorado	2.64	70.6	226	11.9
Graywacke	Alaska	2.77	68.4	221	5.5
Gypsum	Canada	-	-	22	2.4
Limestone	Germany	2.62	63.8	64	4.0
Limestone	Indiana	2.30	27.0	53	4.1
Marble	New York	2.72	54.0	127	11.7
Marble	Tennessee	2.70	48.3	106	6.5
Phyllite	Michigan	3.24	76.5	126	22.8
Quartzite	Minnesota	2.75	84.8	629	23.4
Quartzite	Utah	2.55	22.1	148	3.5
Salt	Canada	2.20	4.6	36	2.5
Sandstone	Alaska	2.89	10.5	39	5.2
Sandstone	Utah	2.20	21.4	107	11.0
Schist	Colorado	2.47	9.0	15	-
Schist	Alaska	2.89	39.3	130	5.5
Shale	Utah	2.81	58.2	216	17.2
Shale	Pennsylvania	2.72	31.2	101	1.4
Siltstone	Pennsylvania	2.76	30.6	113	2.8
Slate	Michigan	2.93	75.9	180	25.5
Tuff	Nevada	2.39	3.7	11	1.2
Tuff	Japan	1.91	76.0	36	4.3

c = cohesion of the rock

ϕ = angle of internal friction

(b) Generally, the shear strength in the laboratory is determined from testing intact rock samples in compression. The value obtained in this way does not take into account the joints and other discontinuities that materially influence the strength behavior of the rock mass.



Linear Failure Envelope Defined by the Equation $\tau = c + \sigma_n \tan \phi$
The stress at a point in a state of incipient failure is represented by the circle through the points representing the minimum principal stress σ_3 and the maximum principal stress σ_1 at that point.

c = cohesion or the rock
ϕ = angle of internal friction of the rock

Figure 8-1. Mohr-Coulomb failure criterion

(7) *Hoek-Brown failure criterion.*

(a) To overcome the difficulties in applying the Mohr-Coulomb theory to rocks, i.e., the nonlinearity of the actual failure envelope and the influence of discontinuities in the rock mass, Hoek and Brown (1980) developed an empirical failure criterion. The Hoek-Brown failure criterion is based on a combination of field, laboratory, and theoretical considerations, as well as experience. It sets out to describe the response of an intact sample to the full range of stress conditions likely to be encountered. These conditions range from uniaxial tensile stress to triaxial compressive stress. It provides the capability to include the influence of several sets of discontinuities. This behavior may be highly anisotropic.

(b) The Hoek-Brown failure criterion is as follows:

$$\sigma_1 = \sigma_3 + \sqrt{m\ \sigma_c\ \sigma_3 + s\ \sigma_c^2}$$

where

σ_1 = major principal stress at failure

σ_3 = minor principal stress at failure

σ_c = uniaxial compressive strength of the intact rock material (given by $\sigma_3 = 0$ and s = 1)

m and *s* are constants that depend on the properties of the rock and the extent to which it has been broken before being subjected to the stresses σ_1 and σ_3.

(c) In terms of shear and normal stresses, this relationship can be expressed as:

$$\tau = (\sigma \times \sigma_3) \sqrt{1 \times m\ \sigma_c / 4\ \tau_m}$$

where

$$\tau_m = 0.5\ (\sigma_1 - \sigma_3)$$

(d) Hoek and Brown (1988) have developed estimates for the strengths of rock masses based on experience with numerous projects. The estimates that cover a wide range of rock mass conditions are given in Table 8-2.

b. In situ stress conditions. The virgin or undisturbed in situ stresses are the natural stresses that exist in the ground prior to any excavation. Their magnitudes and orientation are determined by the weight of the overlying strata and the geological history of the rock mass. The principal stress directions are often vertical and horizontal. They are likely to be similar in orientation and relative magnitude to those that caused the most recent deformations. Some of the simplest clues to stress orientation can be estimated from a knowledge of a region's structural geology and its recent geologic history. Knowledge of undisturbed stresses is important. They determine the boundary conditions for stress analyses and affect stresses and deformations that develop when an opening is created. Quantitative information from stress analyses requires that the boundary conditions are known. Uncertainties are introduced into the analyses by limited knowledge of in situ stresses. Although initial estimates can be made based on simple guidelines, field measurements of in situ stresses are the only true guide for critical structures.

(1) *In situ vertical stress.* For a geologically undisturbed rock mass, gravity provides the vertical component of the rock stresses. In a homogeneous rock mass, when the rock density γ is constant, the vertical stress is the pressure exerted by the mass of a column of rock acting over level. The vertical stress due to the overlying rock is then:

$$\sigma_z = \gamma h$$

Table 8-2
Approximate Relationship Between Rock Mass Quality and Material Constants Applicable to Underground Works

	Carbonate Rocks with Well Developed Crystal Cleavage *dolomite, limestone, and marble*	**Lithified Agrillaceous Rocks** *mudstone, siltstone, shale, and slate (normal to cleavage)*	**Arenaceous Rocks with Strong Crystals and Poorly Developed Crystal Cleavage** *sandstone and quartzite*	**Fine-Grained Polymineralic Igneous Crystalline Rocks** *andesite, dolerite, diabase, and rhyolite*	**Coarse-Grained Polymineralic Igneous and Metamorphic Crystalline Rocks** *amphibolite, gabbro, gneiss, granite, norite, quartz-diorite*
Intact Rock Samples *Laboratory specimens free from discontinuities* RMR = 100, Q = 100	m = 7.00 s = 1.00	10.00 1.00	15.00 1.00	17.00 1.00	25.00 1.00
Very Good Quality Rock Mass *Tightly interlocking undisturbed rock with unweatherd joints at 1 to 3 m* RMR = 85, Q = 100	m = 4.10 s = 0.189	5.85 0.189	8.78 0.189	9.95 0.189	14.63 0.189
Good Quality Rock Mass *Several sets of moderately weathered joints spaced at 0.3 to 1 m* RMR = 65, Q = 10	m = 2.006 s = 0.0205	2.865 0.0205	4.298 0.0205	4.871 0.0205	7.163 0.0205
Fair Quality Rock Mass *Several sets of moderately weathered joints spaced at 0.3 to 1 m* RMR = 44, Q = 1	m = 0.947 s = 0.00198	1.353 0.00198	2.030 0.00198	2.301 0.00198	3.383 0.00198
Poor Quality Rock Mass *Numerous weathered joints at 30-500 mm, some gouge; clean compacted waste rock* RMR = 23, Q = 0.1	m = 0.447 s = 0.00019	0.639 0.00019	0.959 0.00019	1.087 0.00019	1.598 0.00019
Very Poor Quality Rock Mass *Numerous heavily weathered joints spaced < 50 mm with gouge; waste rock with fines* RMR = 3, Q = 0.01	m = 0.219 s = 0.00002	0.313 0.00002	0.469 0.00002	0.532 0.00002	0.782 0.00002

Empirical Failure Criterion:

$$\sigma_1' = \sigma_3' + \sqrt{m\sigma_c\sigma_3' + s\sigma_c^2}$$

σ_1' = major principal effective stress

σ_3' = minor principal effective stress

σ_c' = uniaxial compressive strength of intact rock, and *m* and *s* are impirical constants

CSIR rating: RMR

NGI rating: Q

where γ represents the density that is the unit weight of the rock and generally lies between 20 and 30 kN/m^3.

(2) *In situ horizontal stress.* The horizontal in situ stresses also depend on the depth below surface. They are generally defined in terms of the vertical stress as follows:

$$K_o = \sigma_h/\sigma_v$$

where K_0 represents the lateral rock stress ratio. Since there are three principal stress directions, there will be two horizontal principal stresses. In an undisturbed rock mass, the two horizontal principal stresses may be equal, but generally the effects of material anisotropy and the geologic history of the rock mass ensure that they are not. The value of K_O is difficult to estimate without field measurements. However, some conditions exist for which reasonable estimates can be made. Guidelines for these estimates are as follows:

(a) For weak rocks unable to support large deviatoric stress differences, the lateral and vertical stresses tend to equalize over geologic time. This is called Heim's Rule.

$$\sigma_x = \sigma_y \cong \sigma_z$$

Lithostatic stress occurs when the stress components at a point are equal in all directions and their magnitude is due to the weight of overburden. A lithostatic stress state is widely used in weak geologically undisturbed sediments exhibiting plastic or visco-plastic behavior, such as coal measures, shales, mudstones, and evaporites. It also gives reasonable estimates of horizontal stresses at depths in excess of 1 km.

(b) A lower limiting value of K_O derives from the assumption that the rock behaves elastically but is constrained from deforming horizontally. This applies to sedimentary rocks in geologically undisturbed regions where the strata behave linearly elastically and are built up in horizontal layers such that the horizontal dimensions are unchanged. For this case, the lateral stresses σ_x and σ_y are equal and are given by:

$$\sigma_x = \sigma_y = \gamma \ h \ \nu/(1-\nu)$$

Since Poisson's Ratio for most rocks lies between 0.15 and 0.35, the value of K_O should lie between about 0.2 and 0.55. For a typical rock with a Poisson's Ratio of 0.25, the undisturbed lateral stresses would be 0.33 times the vertical stress. This approach provides a lower bound estimate that applies under appropriate geological conditions.

(c) Amadei, Swolfs, and Savage (1988) have shown that the inclusion of anisotropy broadens the range of permissible values of gravity-induced horizontal stresses in rock masses. For some ranges of anisotropic rock properties, gravity-induced horizontal stresses exceed the vertical stress. Amadei, Swolfs, and Savage have shown that this can be extended to stratified or jointed rock masses.

(d) Residual stresses are the stresses remaining in rock masses after their causes have been removed. During a previous history of a rock mass, it may have been subjected to higher stresses than it is subjected to at the present time. On removal of the load causing the higher stresses, the relaxation of the rock is resisted by the interlocking mineral grains, the shear stresses along fractures, and cementation between particles.

(e) Tectonic stresses are due to previous and present-day straining of the earth's crust. They may arise from regional uplift, downwarping, faulting, folding, and surface irregularities. Tectonic stresses may be active or remnant, depending on whether they are due to present or partially relieved past tectonic events, respectively. The superposition of these tectonic stresses on the gravity-induced stress field can result in substantial changes in both the direction and the magnitude of the resultant primitive stresses. Tectonic and residual stresses are difficult to predict without actual measurement. The evaluation of the in situ state of stress requires knowledge of the regional geology, stress measurements, and observations of the effects of natural stresses on existing structures in rock.

(f) The state of stress at the bottom of a V-shaped valley is influenced by the geometry of both the valley and the hills—the topography.

(3) *In situ stress measurements.*

(a) During the past 20 years, methods for measuring in situ stresses have been developed and a database established. Based on a survey of published results, Hoek and Brown (1980) have compiled a survey of published data that is summarized in Figure 8-2. The data confirm that the vertical stresses measured in the field reasonably agree with simple predictions using the overlying weight of rock.

(b) Horizontal in situ stress rarely show magnitudes as low as the limiting values predicted by elastic theory. The measurements often indicate high stresses that are

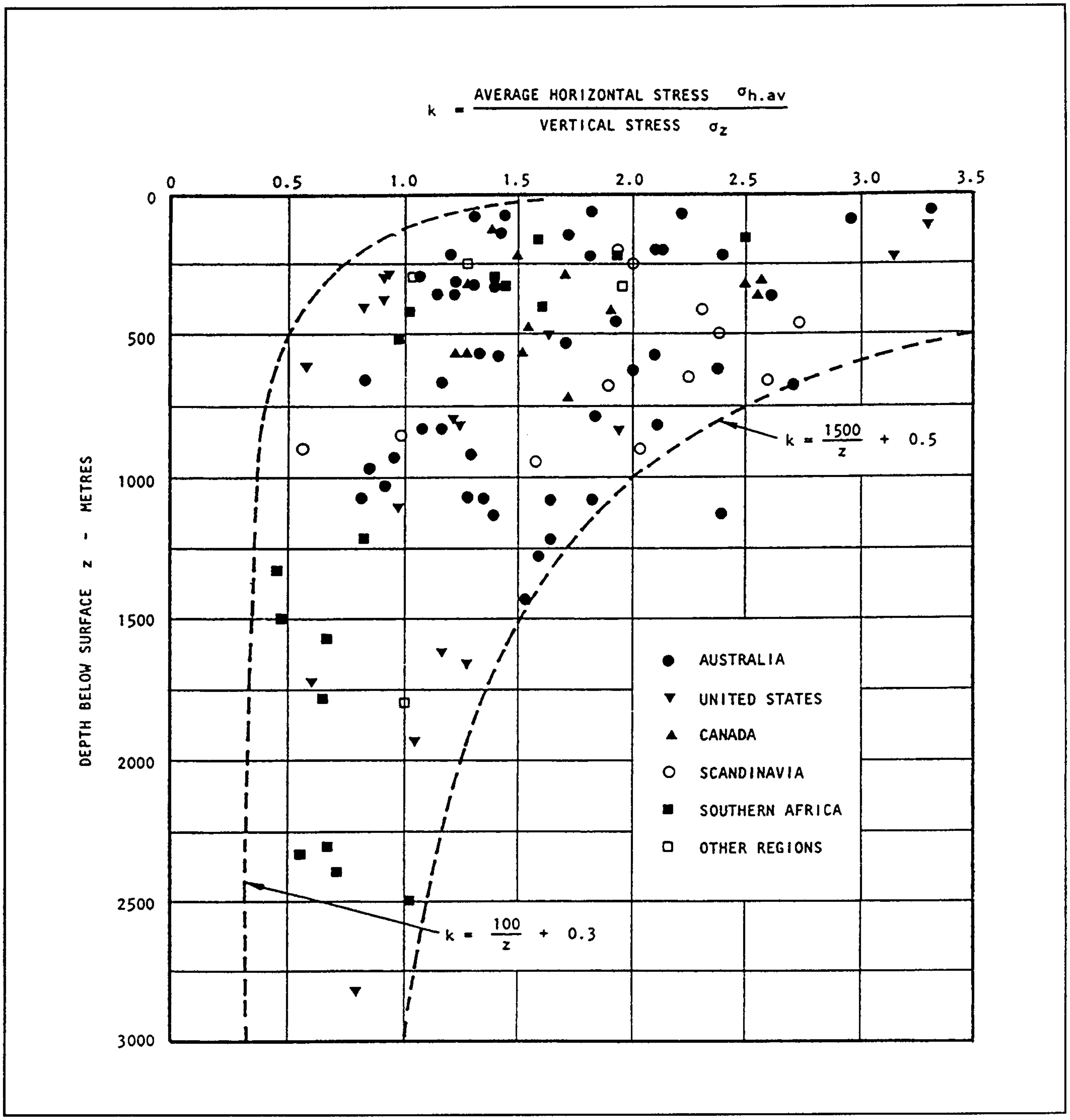


Figure 8-2. Variation of ratio of average horizontal stress to vertical stress with depth below surface

attributed to denudation, tectonics, or surface topography. The horizontal stresses vary considerably and depend on geologic history. At shallow depths, there may be a wide variation in values since the strain changes being measured are often close to the limit of the accuracy of the measuring tools.

8-2. Convergence-Confinement Method

a. The convergence-confinement method combines concepts of ground relaxation and support stiffness to determine the interaction between ground and ground support. As an example, Figure 8-3 illustrates the concept of rock-support interaction in a circular tunnel excavated by a TBM. The ground relaxation curve shown represents poor rock that requires support to prevent instability or collapse. The stages described in Figure 8-3 are outlined below:

b. An early installation of the ground support (Point D_1) leads to excessive buildup of load in the support. In a yielding support system, the support will yield (without collapsing) to reach equilibrium Point E_1. A delayed installation of the support (Point D_2) leads to excessive tunnel deformation and support collapse (Point E_2). The designer can optimize support installation to allow for acceptable displacements in the tunnel and loads in the support.

c. The convergence-confinement method is not limited to the construction of rock-support interaction curves. The method is a powerful conceptual tool that provides the designer with a framework for understanding support behavior in tunnels and shafts. The closed-form solutions (Section 8-3) or continuum analyses (Section 8-4) are convergence-confinement methods as they model the rock-structure interaction. The ground relaxation/interaction curve can also be defined by in situ measurements.

8-3. Stress Analysis

The construction of an underground structure within a rock mass differs from most other building activities. Generally, an aboveground structure is built in an unstressed environment with loads applied as the structure is constructed and becomes operational. For an underground structure, the excavation creates space within a stressed environment. Stress analyses provide insight into the changes in preexisting stress equilibrium caused by an opening. It interprets the performance of an opening in terms of stress concentrations and associated deformations and serves as a rational basis for establishing the performance of requirements for design. The properties of the rock mass are complex, and no single theory is available to explain rock mass behavior. However, the theories of elasticity and plasticity provide results that have relevance to the stress distributions induced about openings and provide a first step to estimating the distribution of stresses around openings. Prior to excavation, the in situ stresses in the rock mass are in equilibrium. Once the excavation is made, the stresses in the vicinity of the opening are redistributed and stress concentrations develop. The redistributed stresses can overstress parts of the rock mass and make it yield. The initial stress conditions in the rock, its geologic structure and failure strength, the method of excavation, the installed support, and the shape of the opening are the main factors that govern stress redistribution about an opening.

a. Excavation configuration and in situ stress state. The excavation shape and the in situ stresses affect the stress distribution about an opening. Since stress concentrations are often critical in the roof and sidewalls of excavations, Hoek and Brown (1980) have determined the tangential stresses on the excavation surface at the crown and in the sidewall for different-shaped openings for a range of in situ stress ratios. They are given in Figure 8-4. These are not necessarily the maximum stresses developing about the opening. Maximum stresses occur at the corners where they can cause localized instabilities such as spalling.

b. Porewater pressures. Stress analysis within the rock mass for tunnelling has been traditionally carried out in terms of total stresses with little consideration given to pore pressures. However, as design approaches for weak permeable rocks are improved, design approaches in terms of effective stress analyses are being developed (Fernandez and Alvarez 1994; Hashash and Cook 1994, see Section 8-4).

c. Circular opening in elastic material. The elastic solution for a deep circular tunnel provides insight into the stresses and displacements induced by the excavation. The tunnel is regarded as "deep" if the free surface does not affect the stresses and displacements around the opening. The problem is considered a plane strain problem and the rock assumed to be isotropic, homogeneous, and linearly elastic. Kirsch's solution (Terzaghi and Richart 1952) disregards body forces and the influence of the boundary at the ground surface. Mindlin's comprehensive solution (1939), which considers the boundary and takes gravity into account, shows that the approximation gives very good agreement for the stresses for depths greater than about four tunnel diameters. Absolute values of stress and

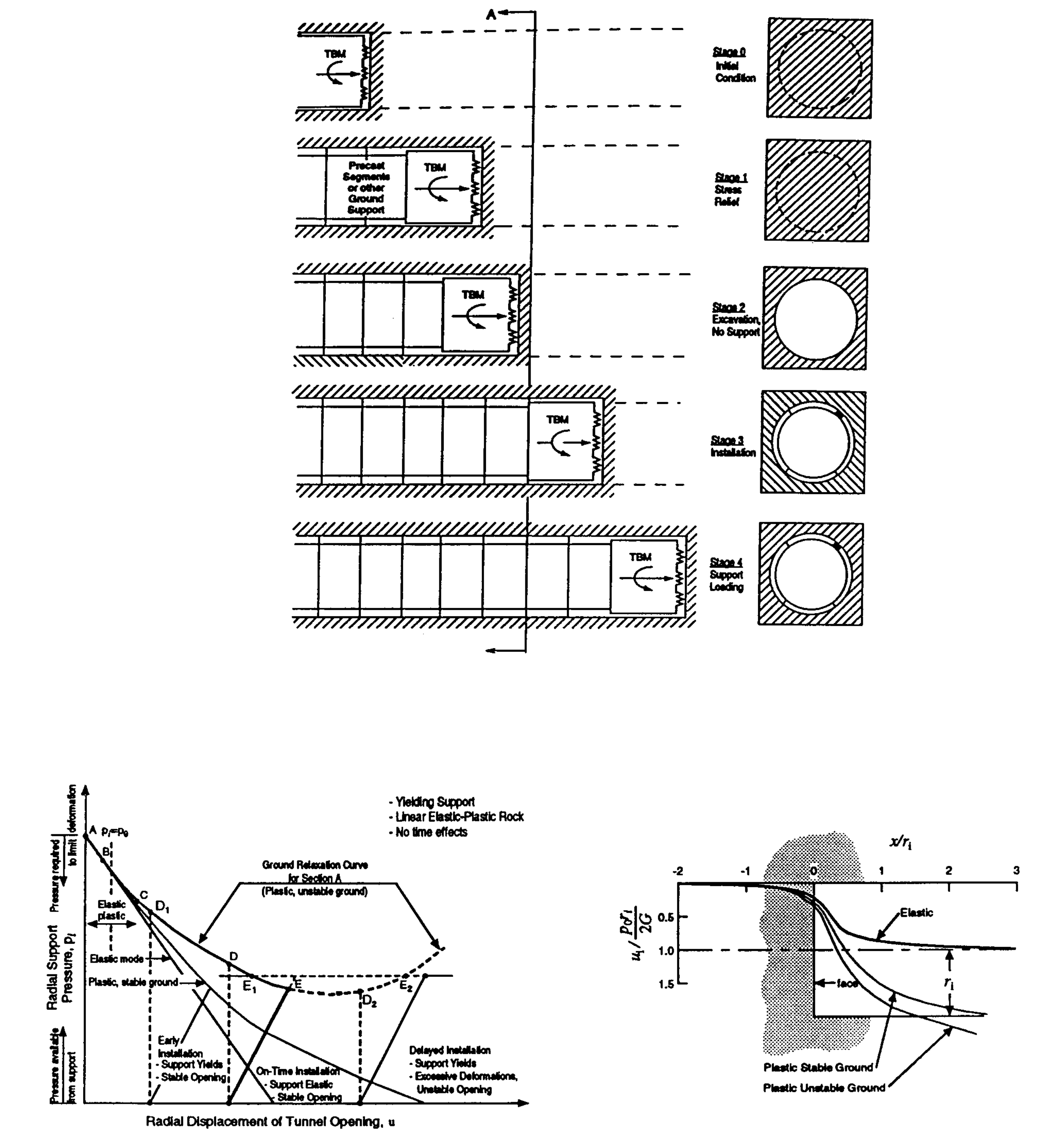
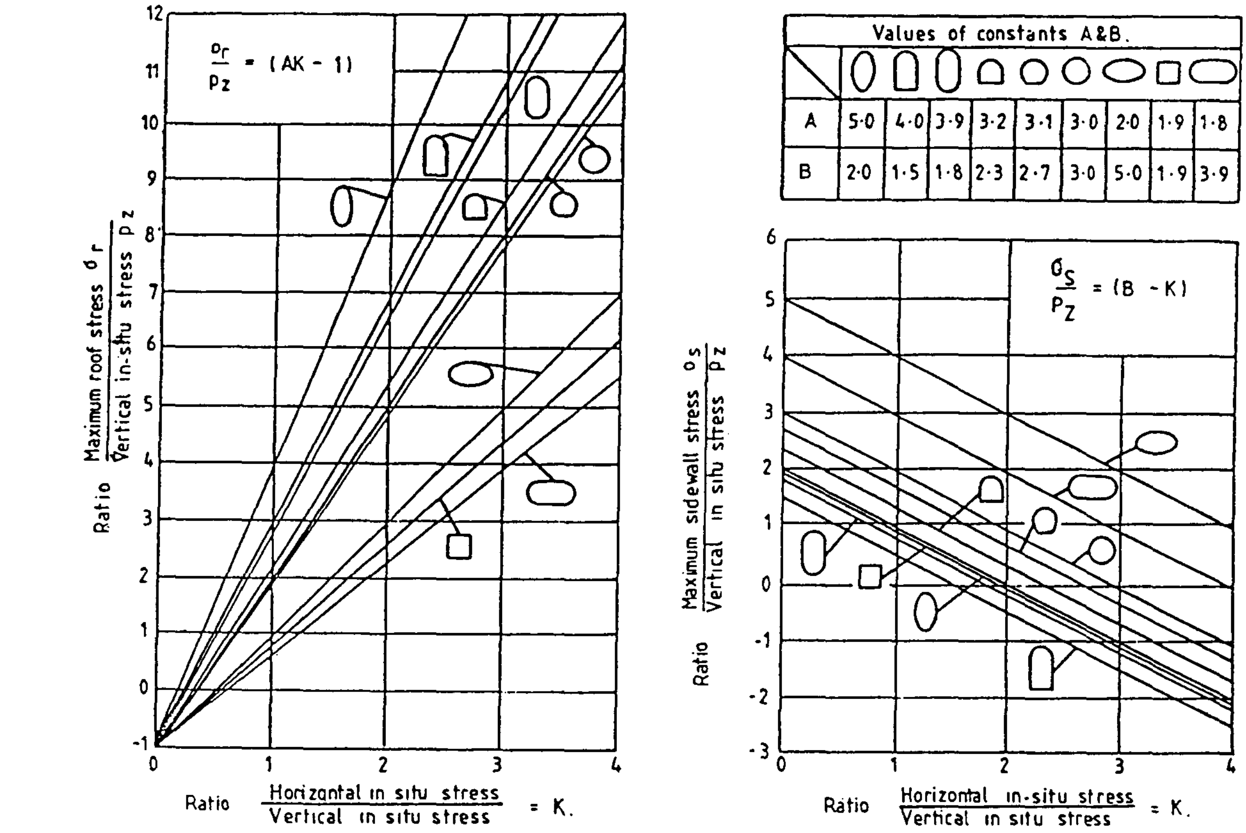


Figure 8-3. Rock-support interaction



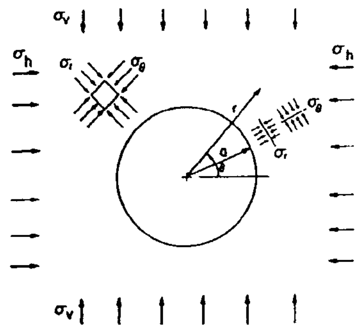
(After Hoek & Brown, 1980)

Figure 8-4. Stresses predicted by elastic analysis

deformation are the same regardless of the sequence of application of loading and excavation; however, relative displacements experienced when the tunnel is driven can only be determined theoretically. Pender (1980) has presented comprehensive solutions for the linear elastic plane strain problem that are summarized in Box 8-1. The simplicity of the elastic solution for the stresses and displacements about a circular opening provides insight into the significance of various parameters and can be used to understand the magnitude of the stresses and deformations induced about an opening.

d. Plastic/yield model. The creation of an underground excavation disturbs the stress field. In the case of weak or even competent rocks subject to high stresses, induced stresses can exceed the strength of the rock leading to its failure. Failure takes the form of gradual closure of the excavation, localized spalling, roof falls, slabbing of side walls, or, in extreme cases, rock bursts. In cases where the violent release of energy is not a factor, this leads to the development of a fractured zone about an excavation that will require stabilization. In strong rocks where brittle or strain softening behavior occurs, strata can be supported relatively easily by the mobilization of the residual strength of the deformed strata by low support pressures. In weaker rocks subject to high stresses where ductile or strain-hardening behavior occurs, possibly over a period of time, much higher restraint is required to support strata; as part of the development of a yield zone, substantial plastic or time-dependent deformations may occur. To estimate these effects, stresses and deformations are

Box 8-1. Stresses Around a Circular Opening in a Biaxial Stress Field



<u>Notation used to describe stresses around a circular opening in a biaxial stress field</u>

a	radius of tunnel shaft
r	radial distance to any point
Θ	angular distance to any point
σ_h, σ_v	original (pre-tunneling) stress field at the tunnel level
σ_θ, σ_r	final (post tunneling) radial and tangential stresses around the tunnel
E	is Young's Modulus of the rock
υ	is the Poisson's Ratio
u_a	is the radial displacement at radius a
v_a	is the tangential displacement at radius a

<u>The stresses are:</u>

radial stress $\sigma_r = 0.5(\sigma_v + \sigma_h)(1 - a^2/r^2) + 0.5\,(\sigma_v - \sigma_h)(1 + 3a^4/r^4 - 4a^2/r^2) \cos 2\Theta$

circumferential stress $\sigma_\theta = 0.5(\sigma_v + \sigma_h)(1 + a^2/r^2) - 0.5\,(\sigma_v - \sigma_h)\,(1 + 3a^4/r^4) \cos 2\Theta$

shear stress $\tau_{r\theta} = 0.5(\sigma_h - \sigma_v)(1 - 3a^4/r^4 + 2a^2/r^2) \sin 2\Theta$

<u>Case 1</u> Stresses applied at a distant boundary - appropriate for condition where a large surface loading is applied after the tunnel is constructed

The displacements are:

$$Eu = (1-\upsilon^2)\{0.5(\sigma_v + \sigma_h)(r + a^2/r) - 0.5(\sigma_v - \sigma_h)(r - a^4/r^3 + 4a^2/r) \cos 2\Theta\} - \upsilon(1 + \upsilon)\{0.5(\sigma_v + \sigma_h)(r - a^2/r) - 0.5(\sigma_v - \sigma_h)(r - a^4/r^3) \cos 2\Theta\}$$

$$Ev = 0.5(\sigma_v - \sigma_h)\,\{\,(1 - \upsilon^2)(r + 2a^2/r + a^4/r^3) + \upsilon(1+\upsilon)(r - 2a^2/r + a^4/r^3)\} \sin 2\Theta$$

At the tunnel periphery, the displacements are:

$$Eu_a = (1-\upsilon^2)a\{(\sigma_v+\sigma_h)-2(\sigma_v - \sigma_h) \cos 2\Theta\}$$

$$Ev_a = 2(1-\upsilon^2)a(\sigma_v - \sigma_h)\sin 2\Theta$$

<u>Case 2</u> Tunnel excavated in a prestressed medium - appropriate for analysis of tunnel excavation

The displacements are:

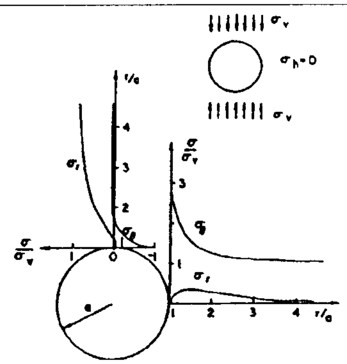
$$Eu = 0.5(1+\upsilon)\{(\sigma_v + \sigma_h)(a^2/r) - (\sigma_v - \sigma_h)((1-\upsilon)4a^2/r - a^4/r^3)) \cos 2\Theta\}$$

$$Ev = 2(1+\upsilon)(\sigma_v - \sigma_h)2a^2/r + a^4/r^3)) \sin 2\Theta$$

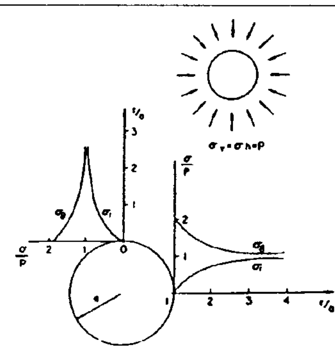
At the tunnel periphery, the displacements are:

$$Eu_a = 0.5(1+\upsilon)a\{(\sigma_v+\sigma_h)-(3-4\upsilon)(\sigma_v-\sigma_h)\cos 2\Theta\}$$

$$Ev_a = 6(1+\upsilon)(\sigma_v-\sigma_h)\sin 2\Theta$$



Radial stress (σ_r) and tangential stress (σ_θ) along the vertical and horizontal axes of a circular tunnel (shaft) in a uniaxial stress field (σ_v).



Radial stress (σ_r) and tangential stress (σ_θ) around a circular tunnel (shaft) in a hydrostatic stress field (P).

calculated from elasto-plastic analyses. The simplest case is that of a circular tunnel driven in a homogeneous, isotropic, initially elastic rock subject to a hydrostatic stress field. The analysis is axisymmetric. The solution assumes plane strain conditions in the axial direction and that the axial stress remains the principal intermediate stress. As the stresses induced by the opening exceed the yield strength of the rock, a yield zone of radius R, develops about the tunnel while the rock outside the yield zone remains elastic. The analysis is illustrated in Boxes 8-2 through 8-5. The rock tends to expand or dilate as it breaks, and displacements of the tunnel wall will be greater than those predicted by elasticity theory. Support requirements are theoretically related to the displacement of the excavation. Deformations are limited by applying a high support pressure, whereas, support pressures are reduced as deformations take place. These theoretical provisions must be tempered with judgment since excessive deformation can adversely affect stability and lead to increased support requirements that are not predicted by the analyses. The elastoplastic solutions for stress distributions and deformations around circular-cylindrical underground openings are summarized in Boxes 8-2, 8-3, and 8-4. It is assumed that the opening is far enough removed from the ground surface that the stress field may be assumed homogeneous and that a lithostatic stress field exists. Body forces are not considered. The assumption is made that the material is either plastic frictionless ($\phi = 0$) or frictional (c-ϕ).

Box 8-2. Elasto Plastic Solution

Reference: Salencon 1969.

$p_z = \sigma_v = \sigma_n$ $\quad$ p_i = Internal Pressure

yield condition: $p_z \geq (p_i + c \cos\phi) / 1 - \sin\phi$

radius of yield zone:

$R = a.[(1 - \sin\phi)(p_z + c.\cot\phi) / (p_i + c\cot\phi)]^{1/(Kp-1)}$

where $K_p = (1 + \sin\phi) / (1 - \sin\phi)$

ELASTIC ZONE:

stresses: $\sigma_r = p_z - (p_z - \sigma_{cp})(R_o/r)^2$

$\sigma_\theta = p_z + (p_z - \sigma_{cp})(R_o/r)^2$

$\sigma_{cp} = p_z(1 - \sin\phi) - c.\cos\phi$ = Radial stress at the Elasto-Plastic interface

deformations: $u_r = (p_z.\sin\phi + c.\cos\phi).(R^2/r)/(2G)$

PLASTIC ZONE:

stresses: $\sigma_r = -c.\cot\phi + (p_i + c.\cot\phi).(r/a)^{Kp-1}$

$\sigma_\theta = -c.\cot\theta + (p_i + c.\cot\phi).K_p.(r/a)^{Kp-1}$

$\sigma_y = (\sigma_r + \sigma_\theta)/2 = c.\cot\phi + (p_i + c.\cot\phi).(1 - \sin\phi)^{-1}.(r/a)^{Kp-1}$

deformations: $u_r = r/(2G).\chi$

where $\chi = (2v-1).(p_z + c.\cot\phi) + (1-v).[(K_p^2 - 1)(K_p + K_{ps})].(p_i + c.\cot\phi).(R/a)^{(Kp-1)}.(R/r)^{(Kps+1)}$

$+ [(1-v).(K_p.K_{ps} + 1)/(K_p + K_{ps}) - v].(p_i + c.\cot\phi).(r/a)^{(Kp-1)}$

and $K_{ps} = (1 + \sin\psi_s)/(1 - \sin\psi s)$ and $G = E/2(1+v)$

Box 8-3. Elasto Plastic Particular Solutions

Particular Solutions to Elastoplastic Problem - c-ϕ Material - Dilation Angle stresses in the elastic and plastic zones are the same as given in Box 8-2.

CASE 1: $\Psi = \phi$, Associated flow rule, $K_p = K_{ps}$
deformations: $u_r = r/(2G).\ \chi$
where $\chi = (2v\text{-}1).(p_z\text{+}c.\cot\phi) + (1\text{-}v).(K_p^2\text{-}1)/(2.k_p)\ .\ (p_i + c.\cot\phi).\ (R/a).^{(Kp\text{-}1)}.(R/r)^{(Kp+1)}$
$+ [(1\text{-}v).(K_p^2\text{+}1)/(2k_p)\ \text{-}v]\ .\ (p_i + c.\cot\phi).(r/a)^{(Kp\text{-}1)}$

CASE 2: $\Psi = 0$, No dilation, $K_p = 1$

deformations: $u_r = r/(2G).\ \chi$

where $\chi = (2v\text{-}1).(p_z\text{+}c.\cot\phi) + (1\text{-}v).(K_p\ \text{-}1).(p_i + c.\cot\phi).\ (R/a).^{(Kp\text{-}1)}.(R/r)^2 + (1\text{-}2.v)\ .(p_i + c.\cot\phi).\ (r/a).^{(Kp\text{-}1)}$

Particular Solutions to Elastoplastic Problem - c-ϕ Material

CASE 3: $\phi = \phi$, and c = 0 , Frictional Material

PLASTIC ZONE:

stresses: $\sigma_r = p_i.(r/a)^{Kp\text{-}1}$

$\sigma_\theta = p_i.K_p.(r/a)^{Kp\text{-}1}$

$\sigma_y = (\sigma_r + \sigma_\theta)/2 = p_i.[(1\text{+}Kp)/2].(r/a)^{Kp\text{-}1}$

deformations: $u_r = r/(2G).\ \chi$

for $\psi = \phi$
$\chi = \quad (2v\text{-}1).p_z\text{+}(1\text{-}v).(K_p^2\text{-}1)/2.K_p.p_i.(R/a).^{(Kp\text{-}1)}.(R/r)^{(Kp+1)} + [(1\text{-}v).(K_p^2\text{+}1)/(2.K_p)\text{-}v].p_i.\ (r/a)^{(Kp\text{-}1)}$

for $\psi = 0$
$\chi = \quad (2v\text{-}1).p_z + (1\text{-}v).(K_p\ \text{-}1).p_i.\ (R/a).^{(Kp\text{-}1)}.(R/r)^2 + (1\text{-}2.v).pi.(r/a).^{(Kp\text{-}1)}$

8-4. Continuum Analyses Using Finite Difference, Finite Element, or Boundary Element Methods

Advances in continuum analysis techniques and the advent of fast, low-cost computers have led to the proliferation of continuum analysis programs aimed at the solution of a wide range of geomechanical problems including tunnel and shaft excavation and construction. For the purpose of this manual, continuum analyses refer to those methods or techniques that assume the rock medium to be a continuum and require the solution of a large set of simultaneous equations to calculate the states of stress and strain throughout the rock medium. The available techniques include the Finite Difference Method (FDM) (Cundall 1976), the Finite Element Method (FEM) (Bathe 1982), and the Boundary Element Method (BEM) (Venturini 1983). While there are subtle advantages of one method over another for some specialized applications, the three methods are equally useful for solving problems encountered in practice. Each of the three numerical techniques is used to solve an excavation problem in a rock medium whereby the field of interest is discretized and represented by a variety of elements. The changes in stress state and deformations are calculated at the element level given the (un)loading (construction) history and material properties. These numerical techniques provide the designer with powerful tools that can give unique insights into the tunnel/shaft support interaction problem during and after construction. Box 8-5 summarizes the steps followed in performing a continuum analysis. The following paragraphs describe these steps and how to consider continuum analyses as part of the design process. Advantages as well as the limitations of the numerical techniques are described.

Box 8-4. Elasto Plastic Particular Solution

Particular Solutions to Elastoplastic Problem

CASE 4: $\phi = 0$, $c = c$

yield condition: $p_z \geq p_i + c$

radius of yield zone: $R = a \,.\, \exp\,[\,(p_z - p_i)/(2.c) - 1/2\,]$

PLASTIC ZONE:

stresses: $\sigma_r = p_i + 2.c.\ \ln(r/a)$

$\sigma_\theta = p_i + 2.c.(1 + \ln(r/a))$

$\sigma_y = (\sigma_r + \sigma_\theta)/2 = p_i + c.(1 + 2.\ln(r/a))$

ELASTIC ZONE:

stresses: $\sigma_r = p_z - c.(a/r)^2 \,.\, \exp\,[(p_z - p_i)/c \ -1]$

$\sigma_\theta = p_z - c.(a/r)^2 \,.\, \exp\,[(p_z - p_i)/c \ -1]$

$\sigma_y = 2.v.p_z$

deformations:

$u_a = \ c\,(1+v).[1 - c\,(1+v)/2.E\,].\ \exp\,[(p_z - p_i)/c \ -1] \cong [c(1+v)/E]\,.\,\exp\,[(p_z - p_i)/c -1]$

Box 8-5. Steps to Follow in Continuum Analysis of Tunnel and Shaft Excavations

1. Identify the need for and purpose of continuum analysis.
2. Define computer code requirements.
3. Modeling of the rock medium.
4. Two- and three-dimensional analyses.
5. Modeling of ground support and construction sequence.
6. Analysis approach.
7. Interpretation of analysis results.
8. Modification of support design and construction sequence, reanalysis.

a. Identify the need for and purpose of continuum analysis. The first step in carrying out a continuum analysis is identifying whether an analysis is needed. The FEM, FDM, or BEM numerical techniques are not substitutes for conventional methods of support design. The support system of a tunnel or shaft opening should first be selected using methods described in Chapters 7 and 9. The continuum analysis is then used to study the influence of the construction sequence and ground deformation on load transfer into supports. Safety factors and load factors commonly used in conventional methods should not be used in numerical analyses. Continuum analyses can incorporate details that cannot be accounted for using conventional methods such as inhomogeneous rock strata and nonuniform initial in situ stress, and hence provide guidance for modifications required in the support system. The continuum methods can best serve to improve support design through the opportunity they provide to study types of situations from which general practical procedures can be developed (e.g., Hocking 1978). Modes of behavior that can be assessed using continuum analysis include the following:

(1) Elastic and elasto-plastic ground/support interaction. Convergence-confinement curves can be constructed using continuum analysis.

(2) Study of modes of failure.

(3) Identification of stress concentrations.

(4) Assessment of plastic zones requiring support.

(5) Analysis of monitoring data.

b. Define computer code requirements. A wide range of commercial and in-house programs are available for modeling tunnel and shaft construction. Prior to performing an analysis using a particular computer code, the user should determine the suitability of the program. Example analyses of problems for which a closed form solution is available (such as those given in Section 8-3) should be performed and the analysis results checked against those solutions. The user should verify that the program is capable of modeling the excavation process correctly and is able to represent the various support elements such as concrete and shotcrete lining, lattice girders, and bolts.

c. Modeling of the rock medium.

(1) The FEM, FDM, and BEM techniques model the rock mass as a continuum. This approximation is adequate when the rock mass is relatively free of discontinuities. However, these methods can still be used to model jointed rock masses by using equivalent material properties that reflect the strength reduction due to jointing (e.g., Zhu and Wang 1993; Pariseau 1993) or a material model that incorporates planes of weakness such as the Ubiquitous Joint Model (ITASCA 1992). Interface elements may be used to model displacements along discontinuities if they are deemed to be an important factor in the behavior of the system. The designer should first use as simple a model as possible and avoid adding details that may have little effect on the behavior of the overall system.

(2) The initial state of stress in the rock mass is important in determining the deformation due to excavation and the subsequent load carried by the support system. In a cross-anisotropic rock mass (in a horizontal topography) where material properties are constant in a horizontal plane, the state of stress can be described by a vertical stress component σ_v due to the weight of rock and a horizontal stress component $\sigma_h = K_0\sigma_v$. K_0 is the lateral in situ stress ratio. In situations where the rock mass is anisotropic, has nonhorizontal strata, or where the ground surface is inclined (e.g., sloping ground), methods such as those proposed by Amadei and Pan (1992) and Pan and Amadei (1993) should be used to establish the initial state of stress in the rock. Such methods are necessary because the initial stresses in the rock mass include nonzero shear stress components.

(3) The choice of a material model to represent the rock medium depends on the available properties obtained from laboratory and in situ testing programs and the required accuracy in the analysis. Many of the available continuum analysis programs have a large material model library that can be used. These include linear elastic and nonlinear elasto-plastic models and may have provisions to incorporate creep and thermal behavior. Available material/constitutive laws for modeling of the rock medium include the following:

- Linear Elastic.
- Non-Linear Elastic (Hyperbolic Model).
- Visco-Elastic.
- Elastic-plastic (Mohr-Coulomb failure criteria with an associated or nonassociated flow rule that controls material dilatancy, Hoek and Brown failure criteria).
- Elastic-viscoplastic.
- Bounding Surface Plasticity (Whittle 1987).

(4) The continuum analysis can be performed assuming either an effective stress or a total stress material behavior. Using effective stress behavior may be more appropriate for use in saturated rock masses and those of sedimentary origin such as shales or sandstones. There is sufficient evidence in the literature that would support the use of the effective stress law for some rocks (e.g., Warpinski and Teufel 1993; Berge, Wang, and Bonner 1993; Bellwald 1992). Examples of effective stress analysis of tunnels can be found in Cheng, Abousleiman, and Roegiers (1993).

(5) The size of the rock field (mesh size) and boundary conditions applied along the far-field edges of the model depend on the size of the opening and the hydrologic conditions. As a rule of thumb, the far-field

boundary is placed at a distance 5-10 times the size of the opening away from the centerline. Pore-pressure boundary conditions along the edges of the model and along the ground surface influence the predicted drawdown condition, pore-pressure buildup, and water inflow into the opening.

d. Two- and three-dimensional analyses. The available numerical techniques can be used to solve a shaft or tunnel excavation problem in two or three dimensions. Two-dimensional (2-D) analysis is appropriate for modeling tunnel sections along a running tunnel. Three-dimensional (3-D) analysis can be useful for understanding the behavior at tunnel and shaft intersections. However, 3-D analyses are laborious and involve the processing of large amounts of data. It is recommended that the analyst use a simplified 2-D model and arrive at a good understanding of the system response before commencing a full blown 3-D analysis. Examples of 2-D and 3-D analyses are given in Box 8-6 and Box 8-7.

e. Modeling of supports and construction sequence. The construction sequence of a tunnel/shaft is complicated and involves many details. It is not practical to incorporate all these details in the numerical simulation. Material removal and liner and dowel installation should be simplified into discrete steps. The following are a few examples of the possible simplifications:

(1) *Tunnel support.* Tunnel support can be cast-in-place concrete, precast concrete segments, shotcrete, or steel sets. The support can be modeled using the same types of elements used to model the rock, but using material models and properties that correspond to the support material. Since the thickness of the support is usually much less than the size of the opening, structural (beam) elements can be used to model the liner. In many situations, these elements are preferred as they better capture the bending behavior of the supports.

(2) *Shotcrete application.* There is usually a lag time between the application of shotcrete and the development of the full strength of the shotcrete. A simple approach to incorporate this effect into the continuum model would be to simulate shotcrete "installation" at the stage when the shotcrete develops its full strength.

(3) *Simulation of transfer of load to tunnel liner in a 2-D analysis.* During tunnel driving, support is installed close to the tunnel face. As the face is advanced, the rock relaxes further and load is applied to the supports. This problem is three-dimensional in nature. In a 2-D model, the rock is allowed to deform a percentage of its otherwise free deformation prior to "installation" of the support. This percentage ranges between 50 and 90 percent (Schwartz, Azzouz, and Einstein 1980) depending on how far the supports are installed behind the tunnel face. Section 8-2 discusses the development of deformations at the tunnel face in the context of the convergence-confinement method.

(4) *Fully grouted dowel with bearing plate.* The principal function of this support element is to reinforce the rock; the bearing plate has a relatively minor role in providing support for the overall system. In the numerical model, the bearing plate can be ignored; only a fully grouted dowel element needs to be represented.

(5) *Simulation of bolts and lattice girders in 2-D analysis.* Bolts and lattice girders are usually installed in a pattern in a tunnel/shaft section and at a specified spacing along the length of the excavation. Therefore, bolts and lattice girders are three-dimensional physical support components. In a 2-D analysis, the properties of bolts and lattice girders are "smeared" along the length of the tunnel. The properties of the bolts and lattice girders used in the model are equal to those of the actual supports averaged by the support spacing along the tunnel/shaft length (i.e., equivalent properties per unit length of tunnel/shaft).

f. Analysis approach. Throughout the process of constructing the model and performing the analyses, it is important to keep the number of details and analyses to a minimum. A well-defined set of parametric studies should be prepared and adjusted as the results of the analyses are examined. The analyst should maintain open communications with the design team. A common mistake is to expect the analysis to provide a resolution or accuracy higher than that of the input data.

g. Interpreting analysis results.

(1) Upon performing the first analysis, the analyst should carefully examine the results. The first step is to check whether the results are reasonable. Some of the questions that should be answered are as follows:

- Is the rock deforming as expected?

- Is the load distribution in the support system consistent with rock deformations?

- Is the change in the state of stress in the rock consistent with the failure criteria and other material properties?

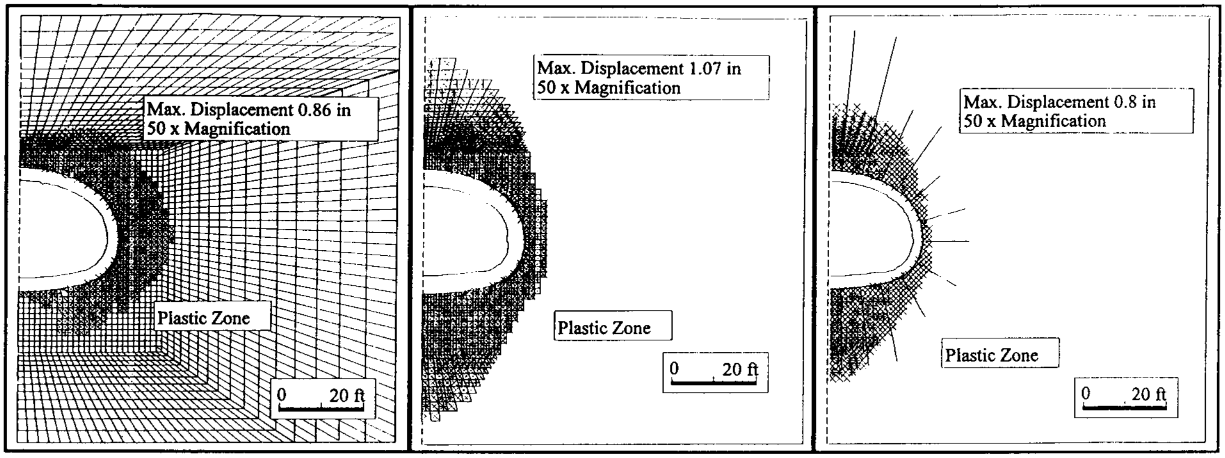
Box 8-6. Two-Dimensional Analysis of Elliptical Tunnel Section

Objective: Study the influence of initial in situ lateral stress ratio, K_0, on deformations and development of plastic zones around an elliptical tunnel section.

Rock Medium: Saturated Taylor Marl Shale, effective cohesion c' = 344 kPa and friction angle Phi' = 30°. Effective stress behavior, elastic-perfectly plastic material with a Mohr-Coulomb failure criteria

Support Type: Unsupported and supported with fully grouted dowels and 10-cm shotcrete lining.

Analysis Type: Finite Difference Analysis (FLAC Program, 2-D)



Deformation and yielded zones, K_o = 1 | **Deformation and yielded zones, K_o = 1.5** | **Deformation and yielded zones, K_o = 1.5**

Analysis Results: The increase in K_0 leads to an increase in the extent of the yielded zones in the crown and invert. Installation of dowels (longer dowels in the crown and invert compared with the springline) and the liner reduces the yielded zone.

Reference: Hashash, Y.M.A., and Cook, R. F. (1994) "Effective Stress Analysis of Supercollider Tunnels," 8th Int. Conf. Assoc. Comp. Methods and Advances in Rock Mechanics, Morgantown, West Virginia.

- Did the solution converge numerically?

Answering these and similar questions might reveal an error in the input data. A detailed check of the numerical results is necessary for the first analysis. A less rigorous check is required for subsequent analyses, but nonetheless the analyst should check for any possible anomalies in the results.

(2) Evaluation of the results of the continuum analyses and their implication regarding the rock-support interaction includes examining the following:

(a) *Deformations around the opening.* Deformations in the rock mass are related to the load transferred to the support system. Data from numerical analyses can be used to develop ground reaction curves (Section 8-2). Parametric studies can be used to develop general design charts that apply to more than one opening size or support configuration.

(b) *Loads in support system.* The analyses can provide moment, thrust, and shear force distributions in the liner. The data provided can be used to address possible modification in the liner, such as the introduction of pin connections to reduce excessive moments. Dowel load data can also be used to revise the distribution and modify the capacity of the proposed dowels. The analyses provide information on the influence of the opening on adjacent structures such as adjacent tunnels or surface buildings that may be distressed due to tunnel/shaft construction. Excessive deformations indicate the need for a more effective support system or a change in the construction method or sequence to mitigate potential damage.

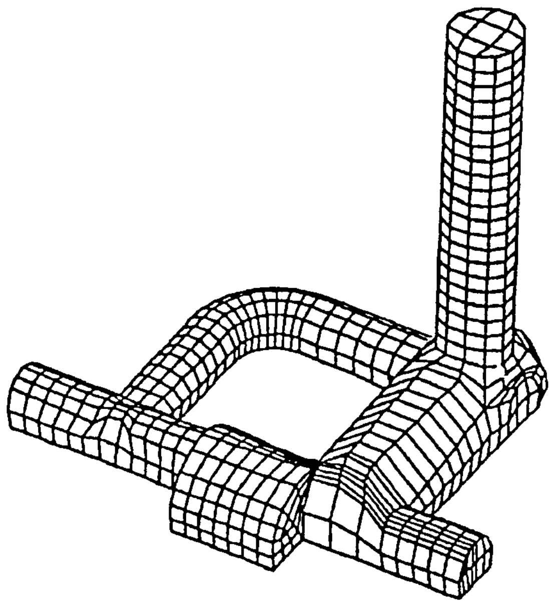
Box 8-7. Three-Dimensional Analysis of a Shaft and Tunnel Intersection

Objective: Study the stress distribution at shaft intersection with tunnel and ancillary galleries

Rock Medium: Eagle Ford Shale overlain by Austin Chalk. Total Stress behavior, linear elastic material

Support Type: No Support

Analysis Type: Finite Element Analysis (ABAQUS Program 3-D)



Analysis Results: Stress concentrations occur at tunnel/shaft intersections at zones experiencing a sudden change in geometry. The extent of the stress concentration is used to estimate the required dowel length in these areas.

Reference: Clark, G.T., and Schmidt, B. (1994) "Analysis and Design of SSC Underground Structures," Proceedings Boston Society of Civil Engineers.

(c) *Yielded and overstressed rock zones.* These zones indicate a potential for rock spalling and rock falls if located near the excavated surface. Large yielded zones indicate a general weakening of the rock and the need to provide reinforcement. The zones can be used to size rock reinforcements (bolts and dowels).

(d) *Pore-pressure distribution and water inflow.* This will provide information on the direction of potential water flow, as well as the expected changes in pore pressures in the rock. The information is relevant in rock masses with discontinuities, as well as in swelling rocks. Contours of pore-pressure distribution are useful in this regard. Many of the commercially available codes have postprocessors that provide the user with a wide range of output capabilities including tabulated data, contour plots, deformed mesh plots, and color graphics. These are useful tools that can convey the results of the analysis in a concise manner especially to outside reviewers.

h. Modification of support system, reanalysis. Continuum analyses provide insight into the behavior of the overall support system and the adequacy of the support system. The analyses may highlight some deficiencies or possible overdesign in the proposed support system. Several analysis iterations may be required to optimize the design.

i. Limitations of continuum analyses. Continuum analysis techniques are versatile tools that provide much understanding of problems involving underground structures. However, they have several limitations that have to be considered to use these techniques effectively. Continuum analysis techniques are not a substitute for conventional design techniques and sound engineering judgement. A continuum analysis cannot give warning of phenomena such as localized spalling. Continuum analysis in geotechnical applications is vastly different from applications in the structural field. Continuum analysis in structural application is geared to satisfy code requirements where the parameters are well defined. Continuum analysis in geotechnical and underground applications involves many unknown factors and requires much judgement on the part of the user. The complexity of a continuum analysis is often limited by the availability of geomechanics data and rock properties. The designer should avoid making too many assumptions regarding the material properties in a model while still expecting to obtain useful information from the analysis. Continuum analyses predict stresses, strains, and displacements but generally do not tell anything about stability and safety factors. Some specialized programs can provide predictions of stability (e.g, Sloan 1981).

j. Example applications. Boxes 8-7 and 8-8 illustrate the use of continuum analyses for shaft and tunnel problems as applied to the Superconducting Super Collider underground structures.

8-5. Discontinuum Analyses

Closed form solutions and continuum analyses of tunnel and shaft problems in rock ignore weaknesses and flaws that interrupt the continuity of the rock mass. The presence of weaknesses makes the rock a collection of tightly fitted blocks. The rock, thus, exhibits a behavior different from a continuous material. This section describes approaches to analysis of openings in rock behaving as a discontinuum.

a. Key block theory.

(1) The best known theory for discontinuous analysis of rocks is the key block theory pioneered by Goodman and Shi (1985). In a key block analysis, the object is to find the critical blocks created by intersections of discontinuities in a rock mass excavated along defined surfaces. The analysis can skip over many combinations of joints and proceed directly to consider certain critical (key) blocks. If these blocks are stabilized, no other blocks can fall into the opening. The principal assumptions are as follows:

(a) All joint surfaces are planar. Linear vector analysis can therefore be used for the solution of the problem.

(b) Joint surfaces extend through the entire volume of the rock mass. No discontinuities terminate within a block. No new discontinuities can develop due to cracking.

(c) The intact blocks defined by the discontinuities are rigid. Deformations are due to block movement but not block deformation.

(d) The discontinuity and excavation surfaces are defined. If the joint set orientations are actually dispersed about a central tendency, one direction must be chosen to represent the set.

(2) Figure 8-5 illustrates the concept of key block analysis. Block analysis can be carried out using stereographic projection graphical methods or vector methods. Hatzor and Goodman (1993) illustrate the application of the analysis to the Hanging Lake Tunnel, Glenwood Canyon, Colorado. The analysis methods have been incorporated into computer programs.

b. Discrete element methods.

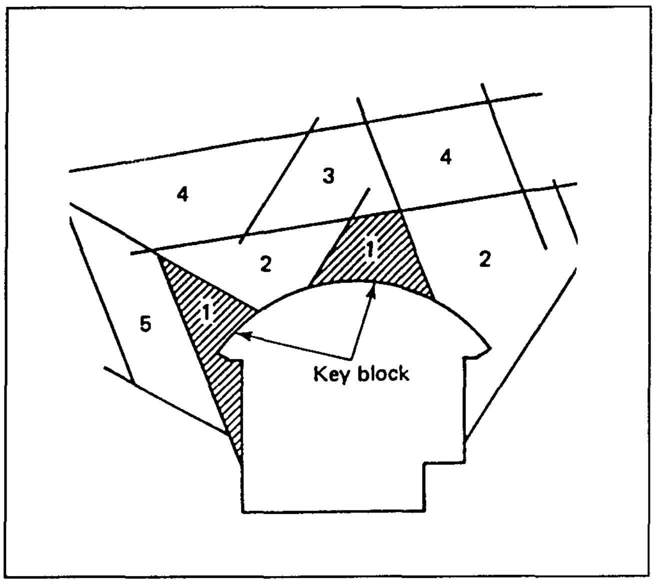


Figure 8-5. Key block analysis

(1) Cundall and Hart (1993) propose that the term discrete element method applies to computer methods that allow finite displacements and rotations of discrete bodies, including complete detachment, and recognize new contacts automatically as the calculation progresses. Four main classes of computer methods conform to this definition:

(a) *Distinct element methods*. They use explicit, time-marching to solve the equations of motion directly. Bodies may be rigid or deformable; contacts are deformable.

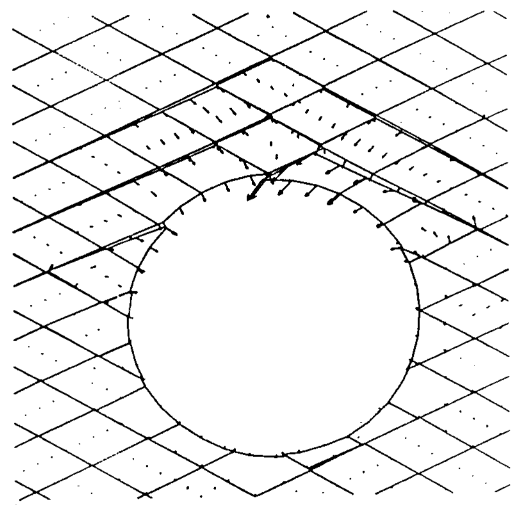
(b) *Modal methods*. They are similar to distinct element methods in the case of rigid bodies, but for deformable bodies, modal superposition is used.

(c) *Discontinuous deformation methods*. In these methods, contacts are rigid, and bodies may be rigid or deformable.

(d) *Momentum-exchange methods*. In these methods, both the contacts and the bodies are rigid; momentum is exchanged between two contacting bodies during an instantaneous collision. Frictional sliding can be represented.

(2) Figure 8-6 shows an analysis of a tunnel opening in a jointed rock mass using the distinct element method and the computer program UDEC.

(3) The block theory and discrete element analysis methods are useful in identifying unstable blocks in large underground chambers. In smaller openings such as shafts and tunnels, they are less useful. Cost considerations may preclude the use of discontinuum analysis in small openings due to budget constraints. Large openings that are used to house expensive equipment have big enough budgets to perform these analyses. Discontinuum analysis methods are limited by the unavailability of sufficient data during design. The methods can be used during construction after mapping of discontinuities to identify potential unstable blocks that require support (NATM).



Movement of blocks around tunnel; velocities denoted by arrows

Figure 8-6. Distinct element analysis, Cundell and Hart 1993